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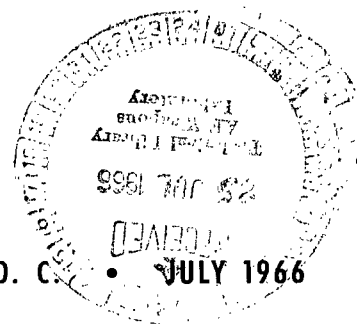
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# THE EQUATORIAL ELLIPTICITY OF THE EARTH AS SEEN FROM FOUR MONTHS OF SYNCOM II DRIFT OVER THE WESTERN PACIFIC

*by C. A. Wagner*

*Goddard Space Flight Center  
Greenbelt, Md.*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1966





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## ABSTRACT

This is the third in a series of reports on the use of tracking information from NASA's Syncom II satellite to define the longitude-dependent gravity field of the earth. This set of orbit data shows that the westward drift of Syncom II decelerated from 0.480 degrees/day at a nodal longitude of  $-174.35^\circ$  in early July 1964, to 0.446 degrees/day at a longitude of  $159.53^\circ$  in late August 1964. Further westward drift of the satellite resulted in an acceleration of the westward drift rate to 0.501 degrees/day when Syncom II reached a mean longitude of  $134.22^\circ$  in early November 1964. On the assumption that this four-month drift of Syncom II was sensitive to only second-order longitude-dependent earth gravity (associated with the ellipticity of the earth's equator), as well as sun, moon and ordinary earth zonal gravity forces, the magnitude and orientation of such equatorial ellipticity is calculated as

$$J_{22} = - (1.86 \pm .14) \times 10^{-6} ,$$

which corresponds to a difference in major and minor equatorial radii of the earth of

$$a_0 - b_0 = 234 \pm 18 \text{ ft} ,$$

and

$$\lambda_{22} = - (16.6 \pm 2.9)^\circ ,$$

which locates the longitude position of the major equatorial axis with respect to Greenwich.

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# THE EQUATORIAL ELLIPTICITY OF THE EARTH AS SEEN FROM FOUR MONTHS OF SYNCOM II DRIFT OVER THE WESTERN PACIFIC

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C. A. Wagner

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## INTRODUCTION

This report is one of a series on the use of the tracking data from Syncom II, the first operational 24-hour satellite, to define the exact nature of the earth's longitude-dependent gravity field. Nineteen orbits for this satellite, calculated from range and range rate observations at the Goddard Space Flight Center, have been used to fix the longitude location for Syncom II during a four-month drift period in the summer and fall of 1964. As in the previous GSFC orbit determinations for Syncom II, longitude-dependent earth gravity was ignored in the calculation of these orbits and a new determination of elements was required about every week to maintain sufficiently accurate position data for the satellite.

Previous reports in this series have covered two three-month "free" drifts of this satellite over Brazil between 55° West and 63° West (Reference 1), and a two-month free drift period over the Central Pacific between 117° West and 163° West (Reference 2). During these long periods between gross orbit change maneuvers initiated by onboard propulsion, this nearly synchronous satellite has remained for sufficient time over limited longitude sectors of the earth to accumulate noticeable drift perturbations caused by small longitude mass anomalies within the earth.

The first gravity experiment with this satellite (Reference 1) tested the sensitivity of its observed accelerated drift as a function of time in a limited longitude region with respect to the simplest kind of longitude-dependent gravity (that associated with an earth whose equator is elliptical, not circular). The second gravity experiment with Syncom II (Reference 2) tested the sensitivity of its observed change of longitude drift rate as a function of mean longitude location over a much longer arc with respect to the same kind of earth gravity.

With minor differences (which could be rationalized as being due to the "noise" of these experiments) these two independent analyses of widely separated Syncom II drift arcs came to the same conclusion—that the earth's longitude gravity field has at least one component (dominant on 24-hour satellites) represented by an elliptical equator whose magnitude gives a second order tesseral constant near

$$J_{22} = -1.7 \times 10^{-6} ,$$

which represents a difference in major and minor axes of about 65 meters or 213 feet, and a major axis orientation near

$$\lambda_{22} = -18^\circ$$

The close agreement of the two previous experimental results appeared to confirm the basic assumption of these analyses, that longitude earth gravity components of higher spacial order than that represented by equatorial ellipticity are of negligible long term influence on the high altitude 24-hour satellite.

This report examines the observed drift rate changes of Syncom II over a *free drift* arc between  $171^\circ$  West and  $133^\circ$  East. This new drift arc was initiated by the pulsing of onboard cold gas jets on 4 July 1964 which resulted in a reduction of Syncom II's westward drift rate from 0.7 degrees/day (at the end of the arc analyzed in Reference 2) to about 0.5 degrees/day.

The analysis of this new arc data for longitude-dependent earth gravity utilizes exactly the same method as developed in Reference 2 and proceeds upon the same basic assumptions as in both previous analyses, namely:

1. Only second-order longitude-dependent earth gravity (associated with an earth whose equator is elliptical) was sensed over the long term drift of Syncom II, in this case from July to November 1964
2. All perturbations on Syncom II not due to gravity during this period were negligible
3. The earth zonal, sun and moon gravity effects on Syncom II drift can be adequately represented over the span of a few months by a small bias in the reduced results from the actual data, which can be estimated from simulated Syncom II trajectories numerically integrated.

The reader may wish to consult References 1 and 2 for further background on the source of the method of analysis used here.

## THEORY OF DATA REDUCTION

The square of the net drift rate of a 24-hour satellite in a second-order longitude gravity field is given (Equation 3 in Reference 2) as

$$(\dot{\lambda})^2 = -A_{22} \cos 2(\lambda - \lambda_{22}) + C_1 \quad (1)$$

where  $C_1$  is a constant determinable from initial or other known or calculable drift conditions. The longitude location of the 24-hour satellite is  $\lambda$ ,  $\lambda_{22}$  is the longitude location of the major axis

of the elliptical equator, and  $A_{22}$  is an earth-gravity-orbit constant given by

$$A_{22} = -72\pi^2 J_{22} \left( \frac{R_0}{a_s} \right)^2 \left( \frac{\cos^2 i_s + 1}{2} \right) \frac{\text{rad}}{(\text{sid day})^2} . \quad (2)$$

In Equation 2,  $J_{22}$  is the second-order longitude gravity constant in the earth's spherical harmonic potential field (Appendix A),  $R_0$  is the mean equatorial radius of the earth and  $a_s$  is the semimajor axis of the nearly circular orbit *synchronous* satellite of orbit inclination  $i_s$ . In Reference 3 it is shown that Equation 1 holds strictly for a synchronous period, circular orbit satellite in an earth field without higher order longitude gravity. For the inclined circular orbit satellite,  $\lambda$  is interpreted as the geographic longitude of the equator crossing. The drift rate of this crossing longitude is  $\dot{\lambda}$ . As long as  $\dot{\lambda}$  is sufficiently small (e.g.,  $\dot{\lambda} < 2$  degrees/day) and the eccentricity of the orbit is not excessive (e.g.,  $e < .01$ ), Equation 1 is still a very good approximation of the law of drift motion for the 24-hour satellite. For somewhat more restrictive mean drift rates and eccentricities than those stated above, the adequacy of Equation 1 has been confirmed by many numerically calculated 24-hour partial drifts as well as by the second integral of the *pendulum* drift motion which stems from Equation 1. The simple order of magnitude arguments of Reference 3 are given in support of the stated probable upper limits on  $\dot{\lambda}$  and  $e$ , for which Equation 1 holds with reasonable accuracy.

For the analysis of the "free" Syncom II drift in July-November 1964 it is merely noted that  $\dot{\lambda} < 0.51$  degrees/day and  $e < 0.001$  applies throughout. To minimize the small error involved in assuming  $A_{22}$  is a constant in the drift, it is assumed that  $A_{22}$  is determined from average values of the relevant orbital parameters ( $i_s$  and  $a_s$ ) over the complete drift period. The arbitrary constant of the motion,  $C_1$ , will be calculated in the analysis as a quasi-mean, constant drift rate for the period about which the longitude-gravity-caused perturbation in the drift rate occurs. Equation 1 can be expanded and rewritten as

$$(\dot{\lambda})^2 = C_1 + C_{22} F_1(\lambda) + S_{22} F_2(\lambda) , \quad (3)$$

where

$$F_1(\lambda) = 36\pi^2 \left( \frac{R_0}{a_s} \right)^2 (\cos^2 i_s + 1) \cos 2\lambda \quad (4)$$

$$F_2(\lambda) = 36\pi^2 \left( \frac{R_0}{a_s} \right)^2 (\cos^2 i_s + 1) \sin 2\lambda \quad (5)$$

$$C_{22} = J_{22} \cos 2\lambda_{22} \quad (6)$$

and

$$S_{22} = J_{22} \sin 2\lambda_{22} . \quad (7)$$

Assume three or more sets of  $\dot{\lambda}$ ,  $\lambda$  values over this free drift arc. Also assume average values for the slightly varying  $a_s$  and  $i_s$  parameters of the orbit. Then, "best" values of the constants  $C_1$ ,  $C_{22}$ , and  $S_{22}$  can be found which minimize, for example, the sums of the squares of the deviations of  $(\dot{\lambda})^2$  (observed) from  $(\dot{\lambda})^2$  as calculated from Equation 3 with this "best set" of drift parameters,  $\hat{C}_1$ ,  $\hat{C}_{22}$ , and  $\hat{S}_{22}$ . This rational data-smoothing procedure is known as "least squares" fitting.

Given the *least squares* values of  $\hat{C}_{22}$  and  $\hat{S}_{22}$ , from Equations 6 and 7 we can calculate "best values" of  $J_{22}$  and  $\lambda_{22}$  from these derived ellipticity parameters as

$$\hat{J}_{22} = -(\hat{C}_{22}^2 + \hat{S}_{22}^2)^{1/2} \quad (8)$$

and

$$\hat{\lambda}_{22} = \frac{1}{2} \tan^{-1} \left( \frac{-\hat{S}_{22}}{-\hat{C}_{22}} \right). \quad (9)$$

The negative signs in numerator and denominator of Equation 9 appear because, from the definition of  $\lambda_{22}$  as the major equatorial axis and the form in which the earth potential function is written in this analysis (Appendix A),  $J_{22}$  *must* be negative. Thus these mandatory negative signs cannot be arbitrarily cancelled if the quadrant of  $\hat{\lambda}_{22}$  is to be properly placed.

## UNSTABLE EQUILIBRIUM LONGITUDE

In Figure 1 it is observed that Syncom II drift in July-November 1964 passes across a longitude  $\lambda_e$  where

$$\left. \frac{d\dot{\lambda}}{d\lambda} \right|_e = 0.$$

Since

$$\frac{d\dot{\lambda}}{d\lambda} = \frac{d\dot{\lambda}}{dt} \frac{dt}{d\lambda} = \frac{\ddot{\lambda}}{\dot{\lambda}}$$

and  $\dot{\lambda}_e$  was finite, this implies  $\ddot{\lambda}_e = 0$ , or  $\lambda_e$  is an equilibrium longitude in the drift regime of this 24-hour satellite. Since the drift rate slowed approaching  $\lambda_e$  (in late August 1964) and increased afterwards,  $\lambda_e$  is evidently a longitude of unstable equilibrium. As such, for the second-order longitude-dependent gravity field assumed in this analysis, it is shown in Reference 1 that  $\lambda_e$  (unstable) is just one of the longitude extensions of the major axis ( $\lambda_{22}$ ) of the elliptical equator.

Thus, this unstable equilibrium longitude (one of four equilibrium longitudes around the equator and an important parameter for the design of operational synchronous satellite systems)



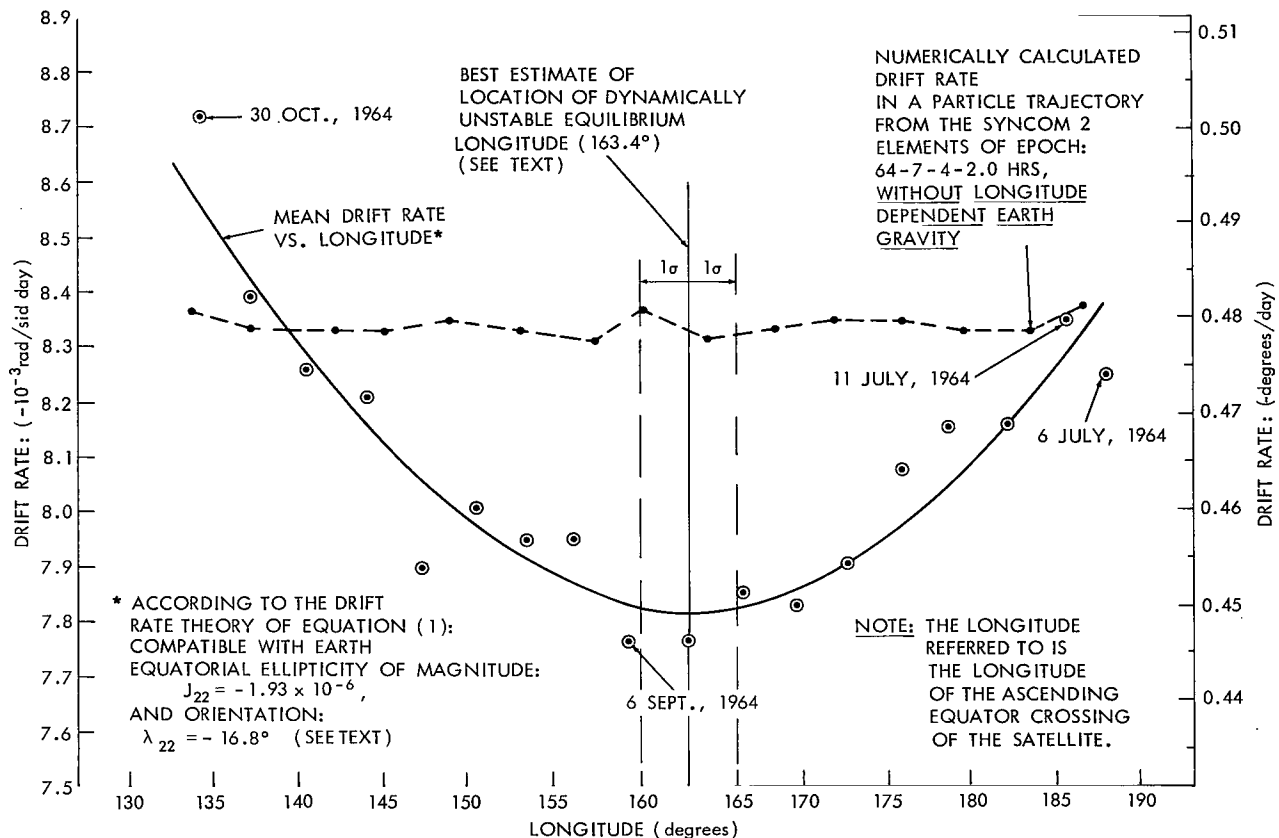


Figure 1—Drift rate of Syncom II (summer and fall, 1964) between  $188^\circ$  and  $134^\circ$  East.

was found directly from  $\hat{\lambda}_{22}$  as measured in this Syncom II - Central Pacific gravity experiment.

## ANALYSIS OF NINETEEN SYNCOM II ORBITS IN THE SUMMER AND FALL OF 1964 TO DETERMINE THE ELLIPTICITY OF THE EARTH'S EQUATOR

Table B-1 in Appendix B lists nineteen "orbits" for Syncom II, about a week apart, which were calculated from returned tracking information on the satellite at Goddard Space Flight Center. These orbits run from 7 July 1964 (when the mean westward drift rate was slowed from 0.7 degrees/day to 0.5 degrees/day at about  $171^\circ$  West) to 2 November 1964, when the equator crossing of the satellite was at about  $133^\circ$  East.\* We can obtain an estimate of the longitude drift rate of the satellite between orbit epochs by simply taking the ratio of the difference in ascending equator crossing longitudes to the difference in time between these near epoch crossings. If this mean

\*Table 1 presents a list of ascending equator crossings for Syncom II nearest the epochs of the GSFC-calculated elements in Appendix B. These have been derived by numerical trajectory generation from these elements using the earth-gravity, sun and moon constants identical to those used in the GSFC orbit determination program.

drift rate between crossings is then assigned to the mean longitude in that interval, a small longitude error will be made which appears to be negligible. If the drift interval is less than about  $10^\circ$  or the time interval less than about 10 days, Appendix C of Reference 2 shows that the error made in this estimate should be negligible. In addition, the efficacy of this *mean* drift and drift rate technique is well proven by the results of the data reduction on the simulated Syncom II trajectory in Appendix C of this report. (In the next section will be found another technique for drift rate estimation which may prove useful in future studies of 24-hour satellite motions.) Table 1 also presents this simply derived drift rate as well as inclination and semimajor axis data at ascending equator crossings for Syncom II in July-November 1964. At the bottom of Table 1 are found average values of  $i_s$  and  $a_s$  in this drift period. An "observation" on the drift rate squared as a function of longitude (according to the theory of Equation 1 or Equation 3) is then taken by the following set of data:

1. the average  $i_s$  for the entire drift period,
2. the average  $a_s$  for the entire drift period,
3. a set of *mean* (bracketed)  $\lambda$  and  $\dot{\lambda}$  values.

With this data  $F_1(\lambda)$  and  $F_2(\lambda)$  can be calculated from Equations 4 and 5 and a condition equation on the "observed"  $(\dot{\lambda})^2$  can be written to satisfy Equation 3. We then assume that each such condition equation on the observed data also contains a small additive and varying amount of error  $\epsilon$  (the so-called residual for that condition equation). The method of least squares estimation of the drift and ellipticity parameters  $C_1$ ,  $C_{22}$ , and  $S_{22}$  finds those parameters  $(\hat{C}_1, \hat{C}_{22}, \hat{S}_{22})$  which minimize the sums of the squares of the  $\epsilon$ 's. These residuals are determined by the differences of  $(\dot{\lambda})_{\text{observed}}^2$  and  $\hat{C}_1 + \hat{C}_{22} F_1(\lambda)_{\text{observed}} + \hat{S}_{22} F_2(\lambda)_{\text{observed}}$  for the complete set of observations. One then can assume for convenience that the residuals are random and normally distributed among the observations with a constant variance  $\sigma_\epsilon^2$  and a mean value zero. In this case it can be shown (Reference 4) that  $\hat{C}_1$ ,  $\hat{C}_{22}$ , and  $\hat{S}_{22}$  are also random and normally distributed with variances proportional to  $\sigma_\epsilon$  and easily calculated from the so called "normal equations" of condition (see also References 2 and 5). The (bracketed) drift and drift rate observations of Table 1 have been inputted into a revised IBM Share program (Reference 6) which sets up and then solves the "normal equations" of condition corresponding to Equation 3.

As outputs in this program (checked for accuracy on the data in Reference 2 which was previously worked up on a desk calculator), estimates of  $\sigma_\epsilon$  (called  $s_\epsilon$ ),  $\sigma(\hat{C}_{22})$  (called  $s(\hat{C}_{22})$ ), and  $\sigma(\hat{S}_{22})$  (called  $s(\hat{S}_{22})$ ), as well as  $\hat{C}_1$ ,  $\hat{C}_{22}$ , and  $\hat{S}_{22}$  have been obtained and are listed at the bottom of Table 1.

The results of this statistical analysis of the drift data on Syncom II (according to the theory of Equation 1 or Equation 3) during July-November 1964 can be summarized by the following least square estimates and standard deviations:

$$\hat{C}_{22} = - (1.6123 \pm 0.1439) \times 10^{-6} \quad (10)$$

$$\hat{S}_{22} = (1.0683 \pm 0.1140) \times 10^{-6}. \quad (11)$$

Table 1

**Syncom II Ascending Equator Crossings and Related Drift Data, Summer and Fall 1964.\***  
**Initial Orbit Epoch (yr-mo-day-hr, universal time): 64-07-04-2.0**

Crossing Number (j)	Date, 1964 (mo-day)	Time, $T_j$ (days from Jan. 0.0, 1964)	$\Delta T_j$ $T_j - T_{j-1}$ (days)	Longitude, $L_j$ (degrees)	$\Delta L_j$ $L_j - L_{j-1}$ (degrees)	Observed Longitude Drift Rate, $\Delta L / \Delta T$ (degrees/day)	Inclination at Crossing, $i_s$ (32.0° data)	Semimajor Axis at Crossing, $a_s$ (6.616 + data $\times 10^{-4}$ earth radii)	Square of Observed Longitude Drift Rate, $Y$ (observed) (rad/sid day) <sup>2</sup>	Square of Computed Longitude Drift Rate, $\hat{Y}$ $Y$ (computed) (rad/sid day) <sup>2</sup>	Square of Residual Longitude Drift Rate, $Y(\text{obs.}) - Y(\text{comp.})$ (rad/sid day) <sup>2</sup>
1	7-6	186.5598		-171.249 (-171.96)		(-.47435)	.541	4.999	6.817 $\times 10^{-5}$	7.056 $\times 10^{-5}$	-2.396 $\times 10^{-6}$
2	7-11	189.5555	2.9957	-172.670 (-174.35)	-1.421	(-.4804)	.537	8.957	6.992	6.893	9.886 $\times 10^{-7}$
3	7-18	196.5455	6.9900	-176.028 (-177.90)	-3.358	(-.4696)	.500	7.147	6.681	6.675	6.146 $\times 10^{-8}$
4	7-25	204.53365	7.98815	-179.7795 (178.82)	-3.7515	(-.4678)	.516	5.120	6.630	6.502	1.279 $\times 10^{-6}$
5	8-1	210.5249	5.99125	177.418 (175.80)	-2.8025	(-.4628)	.500	6.072	6.489	6.369	1.196 $\times 10^{-6}$
6	8-8	217.51435	6.98945	174.183 (172.595)	-3.235	(-.4544)	.476	6.558	6.255	6.258	-2.348 $\times 10^{-8}$
7	8-15	224.5039	6.98955	171.007 (169.44)	-3.176	(-.4498)	.444	2.387	6.129	6.179	-4.950 $\times 10^{-7}$
8	8-22	231.4932	6.9893	167.8635 (166.28)	-3.1435	(-.4520)	.409	4.756	6.1895	6.132	5.791 $\times 10^{-7}$
9	8-29	238.4828	6.9896	164.704 (163.14)	-3.1595	(-.4467)	.443	4.324	6.045	6.117	-7.134 $\times 10^{-7}$
10	9-6	245.4718	6.9890	161.582 (159.53)	-3.122	(-.44635)	.397	2.705	6.036	6.139	-1.031 $\times 10^{-6}$
11	9-13	254.4580	8.9862	157.521 (156.20)	-4.011	(-.4564)	.372	0.392	6.311	6.197	1.140 $\times 10^{-6}$
12	9-19	260.4489	5.9909	154.8365 (153.47)	-2.7345	(-.4571)	.360	4.606	6.330	6.270	5.984 $\times 10^{-7}$
13	9-26	266.4398	5.9909	152.098 (150.49)	-2.7385	(-.4608)	.302	3.414	6.433	6.376	5.645 $\times 10^{-7}$
14	10-3	273.4294	6.9896	148.877 (147.29)	-3.221	(-.4534)	.327	6.017	6.228	6.5195	-2.916 $\times 10^{-6}$
15	10-10	280.4188	6.9894	145.708 (144.055)	-3.169	(-.4730)	.321	4.898	6.778	6.693	8.513 $\times 10^{-7}$
16	10-17	287.4085	6.9897	142.402 (140.51)	-3.306	(-.4749)	.312	7.049	6.830	6.913	-8.370 $\times 10^{-7}$
17	10-24	295.3969	7.9884	138.609 (137.17)	-3.793	(-.48155)	.290	3.824	7.025	7.147	-1.222 $\times 10^{-6}$
18	10-30	301.3880	5.9911	135.724 (134.22)	-2.885	(-.5012)	.250	10.382	7.610	7.373	2.376 $\times 10^{-6}$
19		307.3798	5.9918	132.721	-3.003		.249	5.259			

Average  $i_s = 32.397^\circ$ ; Average  $a_s = 6.6165203$

\*The crossing longitudes without brackets are the first ascending equator crossings of Syncom II after the epoch's of Table B-1 as reported by the Goddard Space Flight Center Tracking and Data Systems Directorate. The longitude drift rates within brackets are the mean longitude rates between successive epoch crossings assumed to occur at a longitude midway between these crossings.

\*\*Computed from Equation 3 using the values of  $\hat{C}_1$ ,  $C_{22}$  and  $\hat{S}_{22}$  below.

Results from the "Least Squares" estimation of the parameters  $C_1$ ,  $C_{22}$  and  $S_{22}$  in Equation 3 for this actual Syncom II drift:

$$\hat{C}_1 = 8.7863 \times 10^{-5} \text{ rad/sid. day}^2$$

$$S(\hat{C}_{22}) = 1.439 \times 10^{-7}$$

$$\hat{C}_{22} = -1.6123 \times 10^{-6}$$

$$S(\hat{S}_{22}) = 1.140 \times 10^{-7}$$

$$\hat{S}_{22} = 1.0683 \times 10^{-6}$$

$$S_r \text{ (standard error of estimate)} = 1.437 \times 10^{-6} \text{ rad/sid. day}^2$$

$$\hat{C}_1 = 8.7863 \times 10^{-5} \frac{\text{rad}}{(\text{sid day})^2} \quad (12)$$

$$S_\epsilon = 1.437 \times 10^{-6} \frac{\text{rad}}{(\text{sid day})^2} \quad (13)$$

Figure 1 presents a graph of the "observed" longitude versus longitude drift rate data for Syncom II in this period together with a theoretical curve calculated from Equation 3 with the above estimates  $\hat{C}_1$ ,  $\hat{C}_{22}$  and  $\hat{S}_{22}$ . In addition, a dashed, broken line gives the results of a numerically calculated (simulated) Syncom II trajectory identical to that in Appendix C except that longitude-dependent earth gravity was not included in the calculation (see Appendix C, Table C-2). The discrepancy between this almost level line and the actual "bowed" data is explained with a fairly reasonable tolerance by the earth's longitude gravity field as represented through Equation 3 by the  $\hat{C}_{22}$  and  $\hat{S}_{22}$  parameters in Equations 10 and 11.

The second-order earth longitude gravity constants  $J_{22}$  and  $\lambda_{22}$  (Appendix A) corresponding to Equations 10 and 11 are calculated from Equations 8 and 9 respectively as:

$$\hat{J}_{22} = - (1.93 \pm 0.13) \times 10^{-6} \quad (14)$$

and

$$\hat{\lambda}_{22} = - (16.8 \pm 2.8)^\circ \quad (15)$$

(See Appendix E of Reference 2 for the treatment of the estimation of the standard deviations of  $\hat{J}_{22}$  and  $\hat{\lambda}_{22}$  from  $\hat{C}_{22}$ ,  $\hat{S}_{22}$  and their standard deviations.)

As the dashed line in Figure 1 shows, the sun, moon and earth zonal gravity fields alone have some small variable influence on the drift rate of Syncom II in this four month period. Therefore, we treat the sensed earth gravity constants in Equations 14 and 15 as unadjusted for these effects (as well as second-order model error due to the approximate nature of Equation 1) and assume they contain a small bias such that:

$$J_{22} \begin{array}{l} \text{(adjusted for sun, moon,} \\ \text{earth zonal and second} \\ \text{order longitude gravity} \\ \text{error)} \end{array} = J_{22} \text{ (unadjusted)} + J_{22} \text{ (bias)} \quad (16)$$

$$\lambda_{22} \text{ (similarly adjusted)} = \lambda_{22} \text{ (unadjusted)} + \lambda_{22} \text{ (bias)} \quad (17)$$

It is noted that we do not make any attempt to adjust over this limited amount of rather noisy data for the presence of an unknown amount of higher order earth longitude gravity (refer to the next section). The simulated Syncom II trajectory in Appendix C with crossing data reasonably close in

time and longitude to the actual Syncom II data provides a means both to test the simple drift rate theory of Equation 1 and to arrive at an estimation of the biases in Equations 16 and 17.

From this simulated Syncom II trajectory (July-November 1964) reduced by the same methods as were used to reduce the actual trajectory (with the sun, moon and other influences treated as *noise*), it is estimated (Appendix C) that the numerically calculated drift was compatible with apparent earth longitude gravity constants  $\hat{J}_{22}$  and  $\hat{\lambda}_{22}$ , where

$$\hat{J}_{22} \text{ (apparent)} = - (1.75 \pm .04) \times 10^{-6} \quad (18)$$

and

$$\hat{\lambda}_{22} \text{ (apparent)} = - (18.2 \pm .6)^{\circ} . \quad (19)$$

Since  $J_{22} = - 1.68 \times 10^{-6}$  and  $\lambda_{22} = - 18.0^{\circ}$  were the actual constants used in the trajectory programs that generated the crossing data, we observe:

1. that accumulated sun, moon, earth zonal and second-order *model* longitude gravity error, on the theory of Equations 1 or 3, is small over the four-month drift period; and
2. that an estimate of the bias of such accumulated effects is contained (from Equations 18 and 19) in Equations 16 and 17 with

$$\begin{aligned} J_{22} \text{ (adjusted)} &= - 1.68 \times 10^{-6} \text{ and } J_{22} \text{ (unadjusted)} = - 1.75 \times 10^{-6} , \\ \lambda_{22} \text{ (adjusted)} &= - 18.0^{\circ} \text{ and } \lambda_{22} \text{ (unadjusted)} = - 18.2^{\circ} , \text{ implying that} \\ \hat{J}_{22} \text{ (bias)} &= - (1.75 \pm 0.04) \times 10^{-6} - (- 1.68 \times 10^{-6}) \\ &= - (0.07 \pm 0.04) \times 10^{-6} , \end{aligned} \quad (20)$$

$$\begin{aligned} \hat{\lambda}_{22} \text{ (bias)} &= - (18.2 \pm 0.6)^{\circ} - (- 18.0^{\circ}) \\ &= - (0.2 \pm 0.6)^{\circ} . \end{aligned} \quad (21)$$

Applying these calculated biases to Equations 14 and 15, we arrive at adjusted values of the actually measured Syncom II gravity constants for the July-November 1964 drift:

$$J_{22} \text{ (measured and adjusted for sun, moon and known earth model effects in Syncom II western Pacific drift)} = - (1.86 \pm 0.14) \times 10^{-6} \quad (22)$$

$$\lambda_{22} \text{ (measured and similarly adjusted)} = - (16.6 \pm 2.9)^{\circ} . \quad (23)$$

The value in Equation 22 corresponds to a difference in major and minor equatorial axes of the geoid of

$$a_0 - b_0 = 233 \pm 17 \text{ feet} . \quad (24)$$

The value in Equation 23 senses the longitude location of the major equatorial axis of the elliptical equator at  $(16.6 \pm 2.9)^\circ$  West.

The discussion covering the theory of data reduction shows that the other extension of the equatorial major axis at  $(163.4 \pm 2.9)^\circ$ , representing a dynamically unstable equilibrium longitude for Syncom II, can be considered to have been directly measured in this gravity experiment. If higher order earth longitude gravity effects are reasonably small as they appear to be (refer to the next and the last sections), this unstable equilibrium longitude measurement will be valid for all 24-hour, near-circular orbit satellites if small epoch-variable sun and moon effects are ignored.

## DISCUSSION

The most gratifying aspect of the results of this straightforward analysis of nineteen new *consecutive* Syncom II updated orbits is, once again, the close agreement with the previous gravity experiments on this satellite over Brazil and the Central Pacific. We have now looked at 54 orbits for Syncom II between  $133^\circ$  East and  $305^\circ$  East longitude for sensitivity to second order earth tesseral gravity. Between  $297^\circ$  East and  $305^\circ$  East the experiment measured longitude gravity constants (Reference 1)

$$J_{22} = - (1.70 \pm 0.05) \times 10^{-6} \quad (25)$$

and

$$\lambda_{22} = - (19.0 \pm 6)^\circ . \quad (26)$$

Between  $245^\circ$  East and  $197^\circ$  East the Syncom II gravity experiment measured (Reference 2)

$$J_{22} = - (1.72 \pm 0.22) \times 10^{-6} \quad (27)$$

and

$$\lambda_{22} = - (17.1 \pm 4.8)^\circ . \quad (28)$$

In this new experiment between  $133^\circ$  East and  $189^\circ$  East the measured constants are

$$J_{22} = - (1.86 \pm 0.14) \times 10^{-6} \quad (29)$$

and

$$\lambda_{22} = - (16.6 \pm 2.9)^\circ . \quad (30)$$

It appears, from a comparison of these numbers with their standard deviations, that one of two conclusions can be supported from the Syncom II study of second order longitude gravity thus far. They are:

1. longitude earth gravity higher than second order has negligible influence on the 24-hour satellite (the second-order gravity constants are roughly  $J_{22} = - (1.8 \pm .15) \times 10^{-6}$  and  $\lambda_{22} = - (18 \pm 4)^{\circ}$ ,  
or
2. longitude earth gravity higher than second order has a small influence on the 24-hour satellite which accumulates about a 10 percent increase in the maximum longitude force over the western Pacific as compared to the maximum force over western Brazilian longitudes.

In view of the study in Reference 7 (see also Figure A1 in Reference 1) on the probable influence of higher order gravity on the Syncom II Brazil measurement, the author is inclined towards the second conclusion above. In that study it is recalled that gravity data from lower altitudes showed unambiguously that the Syncom II Brazil measurement of  $J_{22}$  was likely to have been underestimated by as much as 25 percent. In progress is a comprehensive analysis of all Syncom II and Syncom III drift data thus far which should clarify this situation.

In the study of Syncom II Pacific drift so far, longitude rates have been estimated from differenced crossing data between independently determined "orbits." An alternate approach to the estimation of sets  $\dot{\lambda}$  and  $\lambda$  during the drift of Syncom II has been tried and rejected for this study but may find use as a supplement to the *mean* drift method in future studies of the drift of Syncom II and Syncom III. This method is to use the drift rate shown by the first few observed ascending equator crossings for the satellite after each orbit epoch. In theory the use of such drift rate data would almost double the number of independent observations analyzed in each *free drift* arc. The approach, thus far, has been to use only position data from each orbit. But an orbit is defined (in theory) by independently given velocity as well as position data. Using the first few equator crossings after each epoch as the net drift rate observation for that orbit is equivalent to using the velocity data in that orbit determination. In turn, the accuracy of this approach depends primarily on the accuracy of the semimajor axis determination for Syncom II. Unfortunately, this determination appears to be often too poor to provide sufficient precision to the measurement of drift accelerations in this experiment (see, for example, Figure 1 of Reference 2). Nevertheless, the use of such "short arc" data to supplement the "smoothed" rate data derived from the longer arcs between "orbits" appears promising over a good portion of the Syncom II arcs already studied.

## CONCLUSIONS

Study of the "free" longitude drift acceleration of Syncom II during July-November 1964 over the western Pacific showed sensitivity to the following parameters of earth equatorial ellipticity:

$$J_{22} = - (1.86 \pm 0.14) \times 10^{-6} \quad (31)$$

(corresponding to a difference in major and minor equatorial axes of:  $a_0 - b_0 = 233 \pm 17$  ft), and

$$\lambda_{22} = - (16.6 \pm 2.9)^\circ \quad (32)$$

(locating the geographic longitude of the major equatorial axis).

The above results when compared with previous gravity drift studies of Syncom II appear to be the first measured indication of the small influence of higher order earth longitude gravity on the 24-hour satellite.

Syncom II in its drift over the western Pacific passed over a dynamically unstable equilibrium longitude in the geographic regime of the 24-hour satellite located at

$$\lambda_e = 163.4 \pm 2.9^\circ . \quad (33)$$

These independent experimental results again confirm the conjecture in the first of this series of Syncom II gravity investigations that longitude earth gravity of higher than second order has a small influence on the long term drift of the high altitude 24-hour satellite.

A comprehensive analysis of all Syncom II and III drift data through 1964 is now underway by the author to define the earth's longitude gravity field at 24-hour altitudes for longitudes from  $65^\circ$  to  $305^\circ$  East.

(Manuscript received June 3, 1965)

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10. O'Keefe, J. A., Eckels, A., and Squires, R. K., "The Gravitational Field of the Earth," *Astron. J.* 64(7):245-253, September 1959.
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## Appendix A

### Earth Gravity Potential and Force Field Used in This Report

The gravity potential used as the basis for the data reduction in this study is the exterior potential of the earth derived in Reference A-1 for geocentric spherical coordinates referenced to the earth's spin axis and its center of mass. The infinite series of spherical harmonics is truncated after  $J_{44}$ . The nontesseral harmonic constants  $J_{20}$ ,  $J_{30}$ , and  $J_{40}$  are derived from Reference A-2.

The earth radius  $R_0$  used in this study is

$$R_0 = 6378.388 \text{ km} .$$

The earth's gaussian gravity constant used is

$$\mu_E = 3.9862677 \times 10^5 \text{ km}^3/\text{sec}^2 .$$

Neither of these values, taken from Reference A-3, nor the "zonal geoid" of Reference A-2 is felt to be the most accurate known to date. They are the values used by the GSFC Tracking and Data Systems Directorate to calculate the orbit elements of Syncom II from radar and Minitrack observations. They were chosen to insure consistency between the data of this study and these published orbits, inasmuch as the "triaxial" reduction for which this study has been undertaken is not significantly affected by the probable errors in these values. The second-order tesseral harmonic constants used in the simulation studies were

$$J_{22} = -1.68 \times 10^{-6} ,$$

$$\lambda_{22} = -18 \text{ degrees} .$$

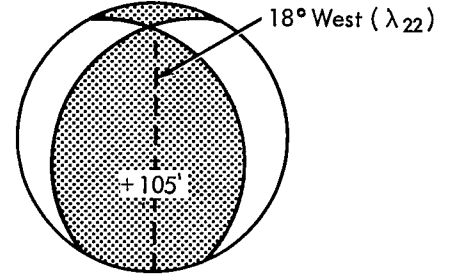
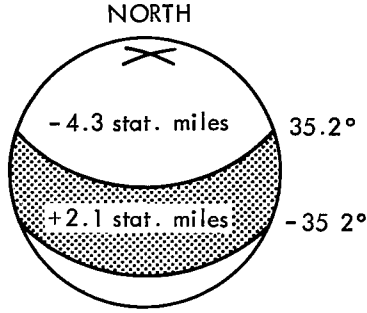
These are the values shown (or implied) on the "tesseral geoid" below (for the  $J_{22}$  harmonic). At a later point in the analysis, the slightly different values reported in the abstract were estimated. The most accurate "zonal geoid" is probably that of Kozai (Reference A-4), with the following earth constants:

$$R_0 = 6378.2 \text{ km} ,$$

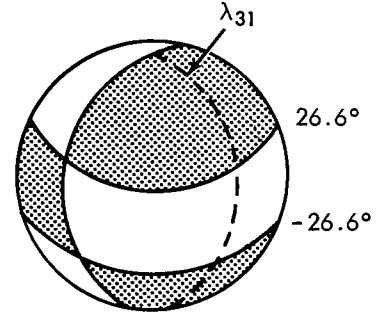
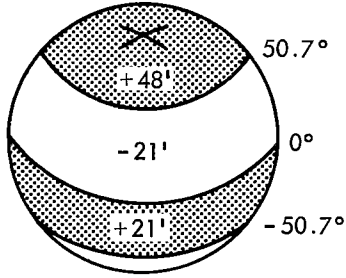
$$J_{22} = 1082.48 \times 10^{-6} , \quad J_{30} = -2.56 \times 10^{-6} , \quad J_{40} = -1.84 \times 10^{-6} ,$$

$$\mu_E = 3.98603 \times 10^5 \text{ km}^3/\text{sec}^2 .$$

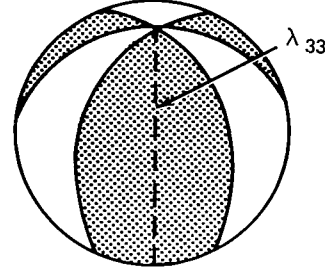
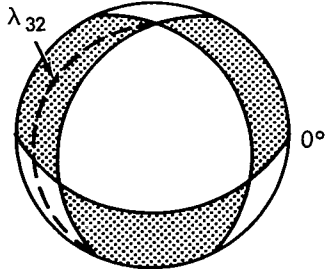
The earth's gravity potential (to fourth order, probably sufficient to account for all significant longitude perturbations on a 24-hour satellite) is given by Equation A-1, which may be illustrated (following Reference A-4, Appendix B, with the zonal constants of Reference A-2) as follows:



$$V_E = \frac{\mu_E}{R} \left[ 1 - \frac{J_{20} R_0^2}{2r^2} (3 \sin^2 \phi - 1) - 3J_{22} \frac{R_0^2}{r^2} \cos^2 \phi \cos 2(\lambda - \lambda_{22}) \right]$$



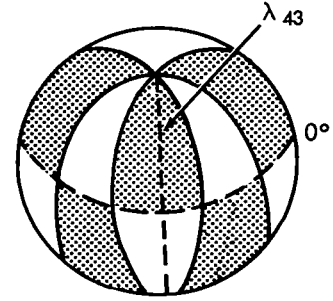
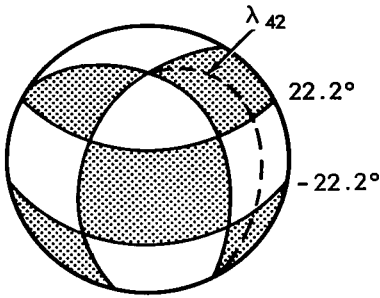
$$- \frac{J_{30} R_0^3}{2r^3} (5 \sin^3 \phi - 3 \sin \phi) - \frac{J_{31} R_0^3}{2r^3} \cos \phi (15 \sin^2 \phi - 3) \cos (\lambda - \lambda_{31})$$



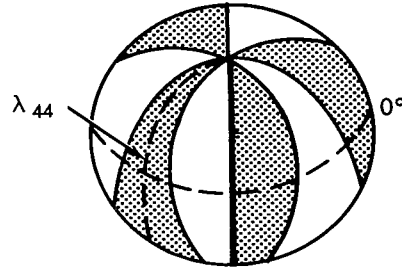
$$- 15J_{32} \frac{R_0^3}{r^3} \cos^2 \phi \sin \phi \cos 2(\lambda - \lambda_{32}) - 15J_{33} \frac{R_0^3}{r^3} \cos^3 \phi \cos 3(\lambda - \lambda_{33})$$



$$- \frac{J_{40} R_0^4}{8r^4} (35 \sin^4 \phi - 30 \sin^2 \phi + 3) - \frac{J_{41} R_0^4}{8r^4} (140 \sin^3 \phi - 60 \sin \phi) \cos \phi \cos (\lambda - \lambda_{41})$$



$$- \frac{J_{42} R_0^4}{8r^4} (420 \sin^2 \phi - 60) \cos^2 \phi \cos 2(\lambda - \lambda_{42}) - \frac{J_{43} R_0^4}{8r^4} 840 \sin \phi \cos^3 \phi \cos 3(\lambda - \lambda_{43})$$



$$- \frac{J_{44} R_0^4}{8r^4} 840 \cos^4 \phi \cos 4(\lambda - \lambda_{44}) \quad (A-1)$$

The earth-gravity field (per unit test mass), whose potential is Equation A-1, is given as the gradient of A-1, or

$$\bar{F} = \hat{r}F_r + \hat{\lambda}F_\lambda + \hat{\phi}F_\phi = \nabla V_E = \hat{r} \frac{\partial V_E}{\partial r} + \frac{\hat{\lambda}}{r \cos \phi} \frac{\partial V_E}{\partial \lambda} + \frac{\hat{\phi}}{r} \frac{\partial V_E}{\partial \phi} : \quad (A-2)$$

or

$$\begin{aligned}
F_r = \frac{\mu_E}{r^2} \left\{ -1 + (R_0/r)^2 \left[ 3/2 J_{20} (3 \sin^2 \phi - 1) + 9 J_{22} \cos^2 \phi \cos 2(\lambda - \lambda_{22}) \right. \right. \\
+ 2(R_0/r) J_{30} (5 \sin^2 \phi - 3) (\sin \phi) + 6(R_0/r) J_{31} (5 \sin^2 \phi - 1) \cos \phi \cos (\lambda - \lambda_{31}) \\
+ 60(R_0/r) J_{32} \cos^2 \phi \sin \phi \cos 2(\lambda - \lambda_{32}) + 60(R_0/r) J_{33} \cos^3 \phi \cos 3(\lambda - \lambda_{33}) \\
+ 5/8 (R_0/r)^2 J_{40} (35 \sin^4 \phi - 30 \sin^2 \phi + 3) \\
+ 25/2 (R_0/r)^2 J_{41} (7 \sin^2 \phi - 3) \cos \phi \sin \phi \cos (\lambda - \lambda_{41}) \\
+ 75/2 (R_0/r)^2 J_{42} (7 \sin^2 \phi - 1) \cos^2 \phi \cos 2(\lambda - \lambda_{42}) \\
\left. \left. + 525 (R_0/r)^2 J_{43} \cos^3 \phi \sin \phi \cos 3(\lambda - \lambda_{43}) + 525 (R_0/r)^2 J_{44} \cos^4 \phi \cos 4(\lambda - \lambda_{44}) \right] \right\}, \quad (A-3)
\end{aligned}$$

$$\begin{aligned}
F_\lambda = \frac{\mu_E}{r^2} (R_0/r)^2 \left\{ 6 J_{22} \cos \phi \sin 2(\lambda - \lambda_{22}) + 3/2 (R_0/r) J_{31} (5 \sin^2 \phi - 1) \sin (\lambda - \lambda_{31}) \right. \\
+ 30 (R_0/r) J_{32} \cos \phi \sin \phi \sin 2(\lambda - \lambda_{32}) + 45 (R_0/r) J_{33} \cos^2 \phi \sin 3(\lambda - \lambda_{33}) \\
+ 5/2 (R_0/r)^2 J_{41} (7 \sin^2 \phi - 3) \sin \phi \sin (\lambda - \lambda_{41}) + 15 (R_0/r)^2 J_{42} (7 \sin^2 \phi - 1) \cos \phi \sin 2(\lambda - \lambda_{42}) \\
+ 315 (R_0/r)^2 J_{43} \cos^2 \phi \sin \phi \sin 3(\lambda - \lambda_{43}) \\
\left. + 420 (R_0/r)^2 J_{44} \cos^3 \phi \sin 4(\lambda - \lambda_{44}) \right\}. \quad (A-4)
\end{aligned}$$

$$\begin{aligned}
F_\phi = \frac{\mu_E}{r^2} (R_0/r)^2 \left\{ -3 J_{20} \sin \phi \cos \phi + 6 J_{22} \cos \phi \sin \phi \cos 2(\lambda - \lambda_{22}) \right. \\
- 3/2 (R_0/r) J_{30} (5 \sin^2 \phi - 1) \cos \phi + 3/2 (R_0/r) J_{31} (15 \sin^2 \phi - 11) \sin \phi \cos (\lambda - \lambda_{31}) \\
+ 15 (R_0/r) J_{32} (3 \sin^2 \phi - 1) \cos \phi \cos 2(\lambda - \lambda_{32}) \\
+ 45 (R_0/r) J_{33} \cos^2 \phi \sin \phi \cos 3(\lambda - \lambda_{33}) - 5/2 (R_0/r)^2 J_{40} (7 \sin^2 \phi - 3) \sin \phi \cos \phi \\
+ 5/2 (R_0/r)^2 J_{41} (28 \sin^4 \phi - 27 \sin^2 \phi + 3) \cos (\lambda - \lambda_{41}) \\
+ 30 (R_0/r)^2 J_{42} (7 \sin^2 \phi - 4) \cos \phi \sin \phi \cos 2(\lambda - \lambda_{42}) \\
+ 105 (R_0/r)^2 J_{43} (4 \sin^2 \phi - 1) \cos^2 \phi \cos 3(\lambda - \lambda_{43}) \\
\left. + 420 (R_0/r)^2 J_{44} \cos^3 \phi \sin \phi \cos 4(\lambda - \lambda_{44}) \right\}. \quad (A-5)
\end{aligned}$$

The actual sea level surface of the earth is to be conceptualized through Equation A-1 as a sphere of radius 6378 km, around which are superimposed the sum of the separate spherical harmonic deviations illustrated. To these static gravity deviations, of course, must be added a centrifugal earth rotation potential deviation at the earth's surface, to get the true sea level surface (Reference A-1).

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- A-3. O'Keefe, J. A., Eckels, A., and Squires, R. K., "The Gravitational Field of the Earth," *Astron. J.*, 64:245-253, 1959.
- A-4. Wagner, C. A., "The Drift of a 24-Hour Equatorial Satellite Due to an Earth Gravity Field Through Fourth Order," NASA Technical Note D-2103, February 1964.





## Appendix B

### Orbital Elements for Syncom II During the Summer and Fall of 1964

Table B-1 gives the elements for Syncom II during July-November 1964 which were derived from radar range and range-rate information on the satellite by the GSFC Data Systems and Tracking Directorate. These elements represent a continuous record of "orbit" updates on Syncom II about every week for this four-month drift period.

Table B-1

Nineteen Consecutive Sets of Syncom II Orbital Elements, Summer and Fall 1964,  
as Reported by the Goddard Space Flight Center.

Epoch, (Yr-Mo-Day-Hr) (Universal Time)	Semimajor Axis, (km)	Eccentricity, $e$	Inclination, $i$ (Degrees)	Right Ascension of the Ascending Node, $\beta$ (Degrees)	Argument of Perigee, $\omega$ (Degrees)	Mean Anomaly, M.A. (Degrees)
64-07-04-2.0*	42202.95	.00086	32.541	312.890	211.723	336.434
64-07-07-3.0	42203.86	.00089	32.537	312.844	210.487	354.275
64-07-13-17.0	42203.01	.00086	32.502	312.801	212.151	205.972
64-07-21-21.0	42202.56	.00095	32.516	312.644	211.691	270.784
64-07-27-16.0	42202.16	.00094	32.502	312.589	215.560	194.968
64-08-03-17.0	42201.94	.00092	32.476	312.455	214.833	214.528
64-08-11-1.0	42201.37	.00090	32.446	312.402	216.021	337.265
64-08-17-19.0	42201.54	.00097	32.409	312.308	215.342	251.656
64-08-25-10.0	42201.16	.00099	32.443	312.262	223.045	113.049
64-09-01-10.0	42201.42	.00087	32.397	312.067	218.832	121.245
64-09-09-14.0	42200.59	.00093	32.373	311.916	218.010	186.557
64-09-15-12.0	42201.51	.00093	32.362	311.863	221.244	156.571
64-09-22-10.0	42201.25	.00096	32.302	311.744	220.581	131.010
64-09-29-6.0	42202.39	.00088	32.327	311.658	221.814	73.456
64-10-06-5.0	42202.54	.00092	32.320	311.589	228.369	55.663
64-10-13-0.0	42203.04	.00094	32.312	311.440	225.495	347.171
64-10-20-16.0	42203.10	.00091	32.289	311.352	226.877	229.785
64-10-26-16.0	42204.92	.00083	32.251	311.193	226.984	232.903
64-11-02-5.0	42202.76	.00090	32.248	311.138	225.015	73.081

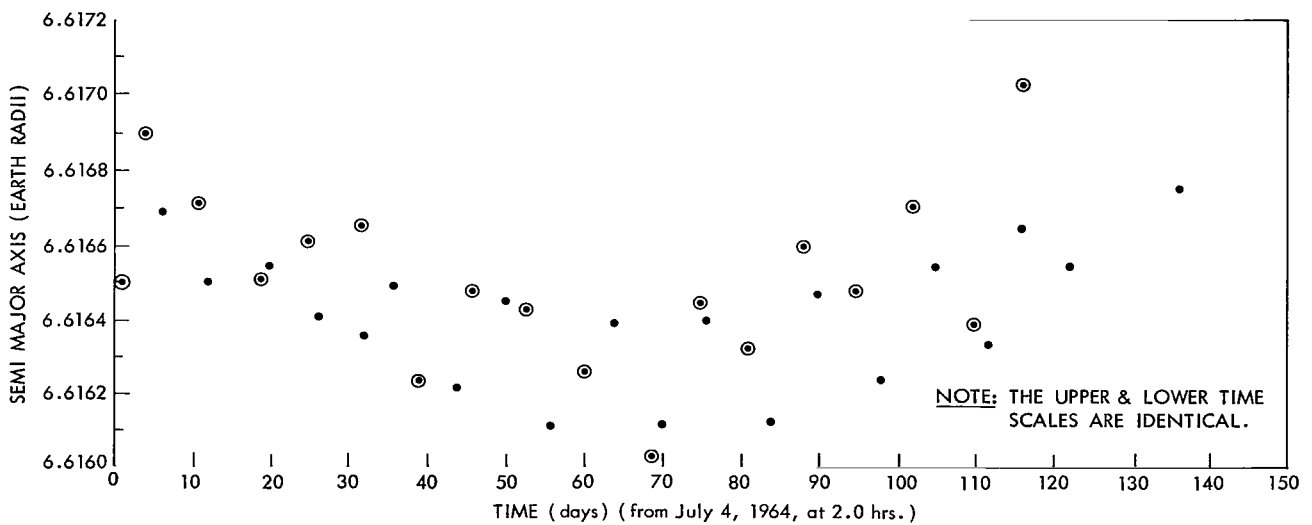
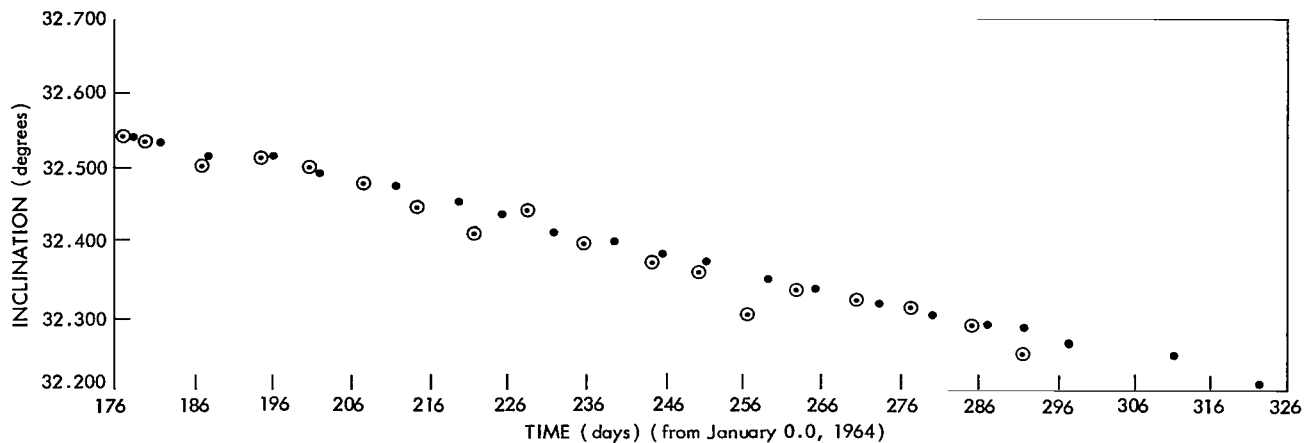
\*This is the first Syncom II epoch after the mean drift rate of the satellite was reduced from -0.7 degree/day to -0.5 degree/day by ground commanded, on-board cold gas jet pulsing on 4 July 1964. The set of eighteen updated orbits which follow are complete to 2 November 1964 to the author's best knowledge. They have been supplied by the Tracking and Data Systems Directorate of the Goddard Space Flight Center through the offices of Dr. Joseph Siry.

By using these elements in the particle trajectory program called "Item" (Interplanetary Trajectory, Encke Method) at GSFC, elements of semimajor axis and inclination and geographic position at the first ascending equator crossing after the epochs of Table B-1 have been calculated.\* The calculated semimajor axes and inclinations at these crossings have been plotted in Figure B-1 along with a similar set from a comparable single particle trajectory calculated by "Item" starting from the elements of Syncom II of 64-07-04-2.0 hours U.T. This latter trajectory program included longitude earth gravity of amount and orientation close to that determined from the actual Syncom II drift data.

In general it can be said that Figure B-1 (and unlisted data on other elements) shows that the actual elements of Syncom II have been reasonably well determined throughout the analyzed drift period, at least for the purpose of this study.

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\*Listed in Table 1, Syncom II Ascending Equator Crossings and Related Drift-Rate Data, Summer and Fall, 1964, of this publication.



- Actual Syncom II Data Estimated by Range and Range Rate Tracking at GSFC
- Orbit Data from a Simulated Syncom II Trajectory Beginning with the Elements of Syncom II (epoch 64-07-04-2.0 hrs.). Determined by the GSFC "Item" Particle Trajectory Program using the Following Earth-Gravity Constants:

$$\begin{aligned} \mu_e &= 3.98627 \times 10^5 \text{ km}^3/\text{Sec}^2 \\ R_o &= 6.38388 \times 10^3 \text{ km/earth radii} \\ J_{20} &= 1082.19 \times 10^{-6} \\ J_{22} &= -1.68 \times 10^{-6} \\ J_{30} &= -2.285 \times 10^{-6} \\ J_{40} &= -2.212 \times 10^{-6} \\ \lambda_{22} &= -18.0^\circ \end{aligned}$$

Figure B-1—Orbit inclination and semimajor axis for Syncom II and a simulated Syncom II trajectory during the summer and fall of 1964.



## Appendix C

### Analysis of Simulated Syncom II Trajectories (Summer and Fall, 1964) for Sensitivity to the Equatorial Ellipticity of the Earth

Table C-1 gives elements and related ascending equator crossing drift data during a simulated Syncom II trajectory computed numerically (by the "Item" program) beginning with the actual elements of Syncom II, epoch 64-07-04-2.0 hours U.T.\* The gravity field of the particle trajectory (including the fields of the sun and the moon) was exactly that used in the orbit determination program for Syncom II, with the addition of equatorial ellipticity introduced into the geoid† of amount and location given by the constants

$$J_{22} = -1.68 \times 10^{-6}$$

and

$$\lambda_{22} = -18^\circ.$$

Table C-2 presents ascending equator crossing drift data from a simulated Syncom II trajectory calculated with the same parameters as that for Table C-1 but without equatorial ellipticity in the geoid model.

The bracketed "mean" longitudes and longitude rates in Tables C-1 and C-2 have been calculated precisely as those in Table 1. A graph of the "observed" (calculated) drift data for the longitude gravity trajectory is shown in Figure C-1.‡

For the longitude gravity crossing data, sensitivity to the drift theory expressed by the equations

$$(\dot{\lambda})^2 = -A_{22} \cos 2(\lambda - \lambda_{22}) + C_1 \quad (1)$$

or

$$(\dot{\lambda})^2 = C_1 + C_{22} F_1(\lambda) + S_{22} F_2(\lambda), \quad (3)$$

\*See Table B-1 in Appendix B.

†See Appendix A for a representation of the geoid used in this simulation.

‡See Figure 1 for trajectory without longitude earth gravity data.

Table C-1

**Ascending Equator Crossings and Related Data in a Simulated Syncom II Trajectory\*  
With Longitude Dependent Earth Gravity, July-November 1964.**

Crossing Number (i)	Date, 1964 (mo-day)	Time, $T_i$ (days from initial epoch)	$\Delta T_i$ $T_i - T_{i-1}$ (days)	Longitude, $L_i$ (degrees)	$\Delta L_i$ $L_i - L_{i-1}$ (degrees)	Observed Longitude Drift Rate, $\Delta L/\Delta T$ (degrees/day)	Inclination at Crossing, $i_s$ (32.0° + data)	Semimajor Axis at Crossing, $a_s$ (6.616 + data $\times 10^{-4}$ earth radii)	Square of Observed Longitude Drift Rate, $Y(\text{observed})$ (rad/sid day) <sup>2</sup>	Square of Computed Longitude Drift Rate, $Y(\text{computed})$ (rad/sid day) <sup>2</sup>	Square of Residual Longitude Drift Rate, $Y(\text{obs.}) - Y(\text{comp.})$ (rad/sid day) <sup>2</sup>
1		1.4751		-171.727			.540	4.743			
	7-7			(-172.68)		(-.4793)			$6.961 \times 10^{-5}$	$6.903 \times 10^{-5}$	$5.805 \times 10^{-7}$
2		5.4693	3.9942	-173.642	-1.915		.539	6.850			
	7-12			(-175.05)		(-.4714)			6.733	6.753	$-2.033 \times 10^{-7}$
3		11.4605	5.9912	-176.466	-2.824		.519	5.038			
	7-19			(181.68)		(-.4644)			6.533	6.565	$-3.152 \times 10^{-7}$
4		19.4487	7.9881	179.824	-3.7095		.512	5.400			
	7-26			(178.45)		(-.4582)			6.361	6.403	$-4.212 \times 10^{-7}$
5		25.4397	5.9910	177.079	-2.745		.493	4.085			
	8-1			(175.72)		(-.4556)			6.288	6.286	$2.234 \times 10^{-8}$
6		31.4306	5.9909	174.350	-2.729		.485	3.561			
	8-6			(173.44)		(-.4550)			6.271	6.202	$6.846 \times 10^{-7}$
7		35.4246	3.9940	172.533	-1.817		.473	5.029			
	8-12			(170.74)		(-.4484)			6.092	6.122	$-3.062 \times 10^{-7}$
8		43.4123	7.9877	168.951	-3.582		.452	2.146			
	8-17			(167.61)		(-.4467)			6.045	6.055	$-9.593 \times 10^{-8}$
9		49.4032	5.99085	166.275	-2.676		.439	4.450			
	8-23			(164.94)		(-.4445)			5.986	6.020	$-3.375 \times 10^{-7}$
10		55.3939	5.99072	163.612	-2.663		.418	1.215			
	8-30			(161.83)		(-.4472)			6.058	6.005	$5.317 \times 10^{-7}$
11		63.3817	7.9878	160.040	-3.572		.399	3.912			
	9-6			(158.71)		(-.4445)			5.987	6.019	$-3.159 \times 10^{-7}$
12		69.3724	5.9907	157.377	-2.663		.381	1.326			
	9-12			(156.04)		(-.4470)			6.053	6.053	$-2.808 \times 10^{-8}$
13		75.3632	5.9908	154.699	-2.678		.372	4.047			
	9-19			(152.91)		(-.4485)			6.094	6.119	$-2.485 \times 10^{-7}$
14		83.3509	7.9877	151.117	-3.583		.346	1.373			
	9-26			(149.75)		(-.4554)			6.283	6.214	$6.904 \times 10^{-7}$
15		89.34185	5.99095	148.3885	-2.728		.336	4.656			
	10-3			(146.57)		(-.4564)			6.312	6.336	$-2.467 \times 10^{-7}$
16		97.3297	7.9879	144.7425	-3.646		.315	2.466			
	10-11			(142.89)		(-.4632)			6.500	6.510	$-9.880 \times 10^{-8}$
17		105.3178	7.9881	141.042	-3.700		.301	5.538			
	10-18			(139.64)		(-.4692)			6.668	6.690	$-2.134 \times 10^{-7}$
18		111.3089	5.9911	138.232	-2.811		.289	3.514			
	10-23			(137.28)		(-.4779)			6.920	6.834	$8.643 \times 10^{-7}$
19		115.3031	3.9942	136.323	-1.909		.286	6.560			
	10-28			(134.89)		(-.4795)			6.966	6.991	$-2.524 \times 10^{-7}$
20		121.2944	5.9913	133.450	-2.873		.268	5.485			
	11-7			(130.02)		(-.4912)			7.309	7.341	$-3.159 \times 10^{-7}$
21		135.2746	13.9802	126.583	-6.867		.250	7.603			

Average  $i_s = 32.401^\circ$ ; Average  $a_s = 6.6164238$  earth radii

## \*Simulated Trajectory Constants

$$\begin{aligned} \mu_e &= 3.98627 \times 10^5 \text{ km}^3/\text{sec}^2 & \lambda_{22} &= -18.0^\circ \\ R_0 &= 63.78388 \times 10^3 \text{ km/earth radii} & J_{30} &= -2.285 \times 10^{-6} \\ J_{20} &= 1082.19 \times 10^{-6} & J_{40} &= -2.12 \times 10^{-6} \\ J_{22} &= -1.68 \times 10^{-6} \end{aligned}$$

The unbracketed longitudes are those printed out from the trajectory program. The bracketed longitude rates are first difference estimates at the bracketed longitudes.

\*\*Computed from Equation 3 using the values of  $\hat{C}_1$ ,  $\hat{C}_{22}$  and  $\hat{S}_{22}$  below. Results from the "Least Squares" estimation of the parameters  $C_1$ ,  $C_{22}$  and  $S_{22}$  in Equation 3 for this simulated trajectory:

$$\begin{aligned} \hat{C}_1 &= 8.4181 \times 10^{-5} \text{ rad/sid. day}^2 & S(\hat{S}_{22}) &= 3.328 \times 10^{-8} \\ \hat{C}_{22} &= -1.4060 \times 10^{-6} & S(\text{standard error of estimate}) &= 4.401 \times 10^{-7} \text{ rad/sid. day}^2 \\ \hat{S}_{22} &= 1.0392 \times 10^{-6} \\ S(\hat{C}_{22}) &= 3.677 \times 10^{-6} \end{aligned}$$

Table C-2

Ascending Equator Crossings and Related Data in a Simulated Syncom II Trajectory  
Without Longitude Earth Gravity, Summer and Fall 1964.\*

Crossing Number (j)	Time (1964) (Mo.-Day)	Time ( $T_j$ ) (Days from initial epoch)	$\Delta T$ , ( $T_{j+1} - T_j$ ) (Days)	Longitude ( $L_j$ ) (Degrees)	$\Delta L$ ( $L_{j+1} - L_j$ ) (Degrees)	Longitude Drift Rate, ( $\Delta L / \Delta T$ ) (Degrees) per day)
1	7-8	1.4751		-171.737		
2		7.4665	5.9914	(-173.2)		(-.4815)
3	7-15	15.4549	7.9884	-174.622	-2.885	
4				(-176.5)		(-.4789)
5	7-23	23.4435	7.9884	-178.448	-3.826	
6	7-31	31.4319	7.9886	(179.6)		(-.4793)
7				177.723	-3.829	
8	8-8	39.4204	7.9884	(175.8)		(-.4797)
9	8-16	47.4088	7.9885	173.891	-3.832	
10				(172.0)		(-.4802)
11	8-24	55.3973	7.9885	170.055	-3.836	
12				(168.1)		(-.4789)
13	9-1	63.3858	7.9884	166.229	-3.826	
14				(164.3)		(-.4782)
15	9-9	71.3741	7.9885	162.409	-3.820	
16				(160.5)		(-.4812)
17	9-17	79.3626	7.9885	158.565	-3.844	
18				(156.6)		(-.4782)
19	9-25	87.3510	7.9883	154.745	-3.820	
20				(152.8)		(-.4786)
21	10-3	95.3394	7.9885	150.922	-3.823	
22				(149.0)		(-.4801)
23	10-11	103.3278	7.9884	147.087	-3.835	
24				(145.2)		(-.4788)
25	10-19	111.3162	7.9884	143.262	-3.825	
26				(141.3)		(-.4792)
27	10-27	119.3047	7.9828	139.434	-3.828	
28				(137.5)		(-.4787)
29				135.610	-3.824	
30				(133.7)		(-.4811)
31				131.767	-3.843	

\*Initial Orbit Epoch of the Simulated Trajectory (yr.-mo.-day-hr, universal time) 64-07-04-2.0.

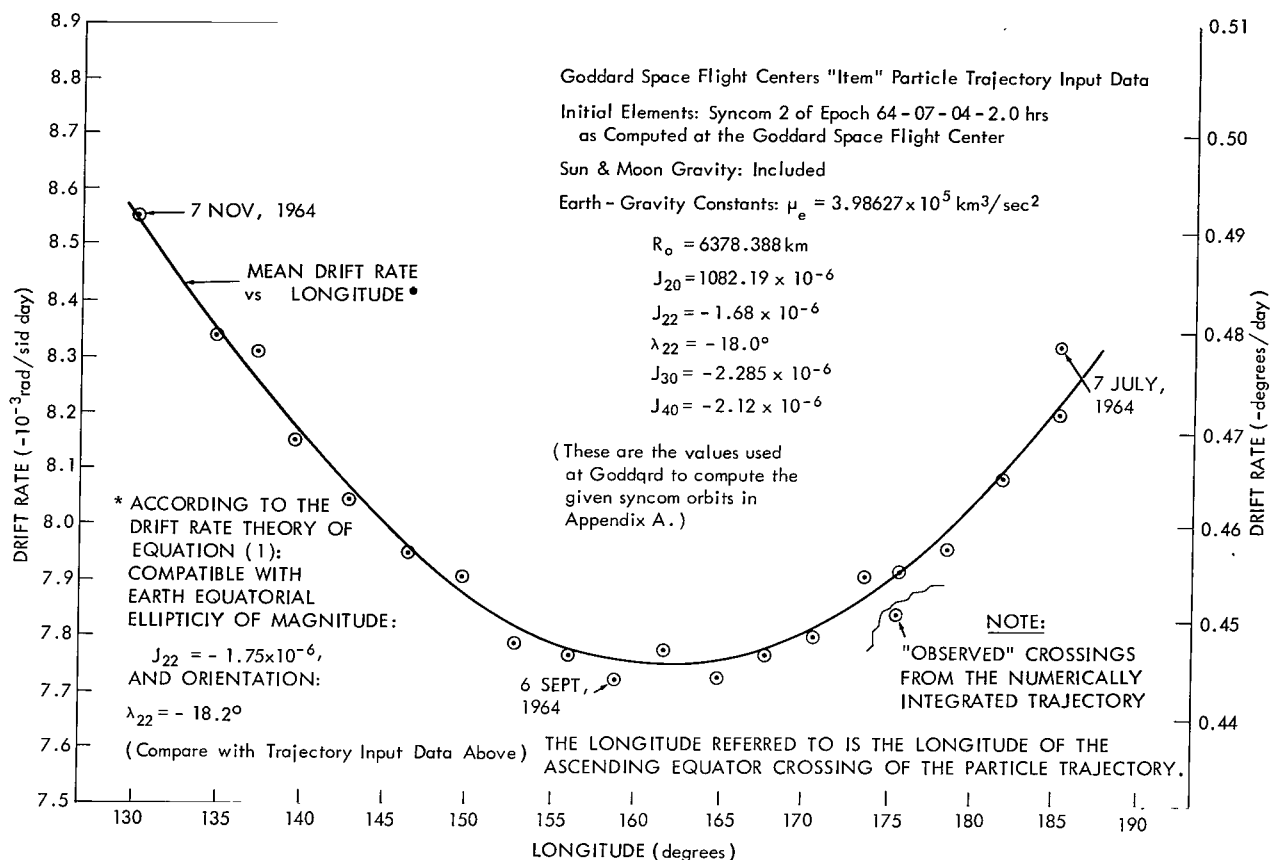


Figure C-1—Drift rate in a simulated Syncom II trajectory between 187° and 130° East, summer and fall, 1964.

has been assessed by a revised multiple regression IBM "Share" program\* in exactly the same manner as the sensitivity of the actual Syncom II data has been tested. Here the "noise sources" in the experiment are due just to sun, moon, and earth zonal gravity unaccounted for by second-order tesseral earth gravity as well as errors arising from the fact that Equation 3 is an approximation of the actual drift perturbation due to such second order longitude gravity.

The results of this regression analysis (for  $C_1$ ,  $C_{22}$ ,  $S_{22}$  in Equation 3) on the longitude gravity simulated trajectory are summarized in Table C-1 as:

$$\hat{C}_1(\text{sim.}) = 8.4181 \times 10^{-5} \frac{\text{rad}}{(\text{sid day})^2} \quad (\text{C-1})$$

$$\hat{C}_{22}(\text{sim.}) = -1.4060 \times 10^{-6} \quad (\text{C-2})$$

$$\hat{S}_{22}(\text{sim.}) = 1.0392 \times 10^{-6} \quad (\text{C-3})$$

\*Kunin, M. J., "Snap-7090 Multiple Regression Analysis Program," IBM 7090 Program No. 183, "Share" Distribution Number 1289, New York: Shell Oil Company Data Processing Department, January 3, 1962.



$$s(\hat{C}_{22, sim.}) = 3.677 \times 10^{-8} \quad (C-4)$$

$$s(\hat{S}_{22, sim.}) = 3.328 \times 10^{-8} \quad (C-5)$$

$$s_e(\text{standard error of estimate, simulated}) = 4.401 \times 10^{-7} \quad (C-6)$$

It is noted that  $s_e$  (simulated) in this drift period is about a third of  $s_e$  (actual) during the same period and over about the same number and location of drift rate "observations".\* But  $s_e$  (sim.) in the previous Syncom II drift period analyzed (Reference C-1) was only about two-thirds of  $s_e$  (actual) in that period. The greater amount,  $s_e$  (actual), in both periods was roughly the same. But  $s_e$  (sim.) for the earlier drift period was twice what it is in this new study. It appears likely then that the actual "observation" error in this new drift period is somewhat greater in this new four-month period than such "position" error in the old. This is the error in the time and crossing data supplied by the GSFC Tracking and Data Systems Directorate, as distinguished from sun, moon, and other second order gravity noise. Using the values from Equations C-2 through C-4 in the equations

$$\hat{J}_{22} = -(\hat{C}_{22}^2 + \hat{S}_{22}^2)^{1/2} \quad (C-7)$$

and

$$\hat{\lambda}_{22} = \frac{1}{2} \tan^{-1} \left( \frac{-\hat{S}_{22}}{-\hat{C}_{22}} \right), \quad (C-8)$$

it is found that

$$\hat{J}_{22}(\text{sim.-calculated}) = J_{22}(\text{apparent, in the simulated trajectory}) = -(1.75 \pm 0.04) \times 10^{-6} \quad (C-9)$$

and

$$\hat{\lambda}_{22}(\text{sim.-calculated}) = \lambda_{22}(\text{apparent, in the simulated trajectory}) = -(18.2 \pm 0.6)^\circ \quad (C-10)$$

Since

$$J_{22}(\text{actual, in the simulation}) = -1.68 \times 10^{-6}$$

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\*See Equation 13 in this publication.

and

$$\lambda_{22} \text{ (actual, in the simulation) } = -18^\circ,$$

it is apparent that the error biases due to sun, moon and second order gravity "noise" inherent in the use of Equation 3 to approximate the longitude gravity drift in this 24-hour satellite arc are acceptably small. The good experience with these closely simulated trajectories in References C-1 and C-2, as well as here, suggests the justification of utilizing the simulated results to predict the likely small, accumulated bias in the actual Syncom II data reduction from these most well-known or calculable error sources.\* Other techniques of removing these calculable errors point for point, instead of in this comprehensive manner (for a single drift arc), are being explored.

#### REFERENCES

- C-1. Wagner, C. A., "The Equatorial Ellipticity of the Earth from Two Months of Syncom II Drift over the Central Pacific," NASA Technical Note D-3315, 1966.
- C-2. Wagner, C. A., "Determination of the Ellipticity of the Earth's Equator from Observations on the Drift of the Syncom II Satellite," NASA Technical Note D-2759, May, 1965.

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\*This was done in Equations 20 and 21 in this publication.

## Appendix D

### List of Symbols

$A_{22}$	Second-order gravity-drift constant associated with the drift regime of a 24-hour satellite.
$C_{22}$	Cosine parameter of earth-equatorial ellipticity.
$\hat{C}_i$	"Best" or "least-squares" estimation of the parameter $C_i$ .
$J_{22}$	Second-order tesseral constants of the earth's gravity field (associated with magnitude and geographic orientation of equatorial ellipticity).
$R_0$	Mean equatorial radius of the earth.
$S_{22}$	Sine parameter of earth-equatorial ellipticity.
$a_0$	Major equatorial radius of the earth.
$a_s$	Semimajor axis of the orbit of the near-synchronous satellite.
$b_0$	Minor equatorial radius of the earth.
$e$	Orbit eccentricity; when subscripted, equilibrium.
$i_s$	Inclination of the orbit of the near-synchronous satellite.
$s( )$	Estimate of standard deviation of random variable ( ).
$\epsilon_i$	Random error associated with observation $i$ .
$\lambda$	Geographic longitude of the ascending equator crossing.
$\sigma_\epsilon^2$	Constant variance of $\epsilon_i$ (estimate of $\sigma_\epsilon$ is called $S_\epsilon$ )

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